Math 70 4.4 Applications of Linear Systems (2x2) 9.2 - 1st (3x3) Lesson 12

4.4 Applications of Systems of Linear Equations (2x2)

Objectives

1) Solve "total value" problems (mixture w/ money)

2) Solve mixture problems

3) Solve uniform motion problems (D=RT)

9.2 Applications of Systems of Linear Equations (3x3)

1) Direct translation

2) Mixture

3) Total value

{ Lesson 13 is EXAM 1 chapters 1-2-3 }

Math 70 4.4 & 9.2 Problem Solving with Systems of Linear Equations

Objectives:

1) Solve problems that can be modeled by a system of two linear equations.
2) Solve problems with cost and revenue functions
3) Solve problems that can be modeled by a system of three linear equations.

1) Two cars leave Indianapolis, one traveling east and the other west. After 3 hours they are 297 miles apart. If one car is traveling 5 mph faster than the other, what is the speed of each?

2) Terry can row 10.6 km in 3 hour downstream and 6.8 km upstream in 2 hour. Find how fast Terry can row in still water and the speed of the current.

3) Lynn Pike, a pharmacist, needs 70 liters of a 50% alcohol solution. She has available a 30% alcohol solution and an 80% alcohol solution. How many liters of each solution should she mix to obtain 70 liters of a 50% solution?

4) A pharmacist needs 500 ml of a 20% Phenobarbital solution but has only 5% and 25% Phenobarbital solutions available. Find how many ml of each he should mix to get the desired solution.

5) Rabbits in a lab are kept on a strict daily diet that includes 30 g of protein, 15 g of fat, and 24 g of carbohydrates. The scientist has only three food mixes available with the grams of nutrients per unit given by the table below. Find how many units of each mix are needed daily to meet each rabbit's dietary need. Round to the nearest tenth if necessary.

	protein	fat	carbohydrate
Mix A	4	6	3
Mix B	6	1	2
Mix C	4	1	12

6) A manufacturing company recently purchased \$3000 worth of new equipment to offer new personalized stationery to its customers. The cost of producing a package of personalized stationery is \$3.00, and it is sold for \$5.50. Find the number of packages that must be sold for the company to break even.

7) The measure of the largest angle of a triangle is 80° more than the measure of the smallest angle, and the measure of the remaining angle is 10° more than the measure of the smallest angle. Find the measure of each angle.

8) A drafting student bought three templates and a pencil for \$6.45, then went back and bought two pads of paper and four pencils for \$7.50. If the price of a pad of paper is three times the price of a pencil, find the prices of each type of item.

Extras:

2 variable problems

1. Find how many quarts of 4% butterfat milk and 1% butterfat milk should be mixed to yield 60 quarts of 2% butterfat milk.
2. Karen bought some large frames for \$15 each and some small frames for \$8 each at a closeout sale. If she bought 22 frames for \$239, find how many of each type she bought.
3. An office supply store sells 7 writing tablets and 4 pens for \$6.40. Also, 2 tablets and 19 pens cost \$5.40. Find the price of a tablet and the price of a pen.
4. A candy shop manager mixes M&M's worth \$2 per pound with trail mix worth \$1.50 per pound. Find how man pounds of each she should use to get 50 pounds of a party mix worth \$1.80 per pound.
5. Two cyclists start at the same point and travel in apposite directions. One travels 4 mph faster than the other. In 4 hours they are 112 miles apart. Find how fast each is traveling.
6. Find the break-even point if $C(x) = 105x + 70{,}000$ and $R(x) = 245x$.

3 variable problems:

7. The sum of three numbers is 40. One number is 5 more than a second number. It is also twice the third. Find the numbers.
8. The perimeter of a triangle is 92 cm. If two sides are equally long and the third side is 9 cm longer than the others, find the lengths of the three sides.

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1) Two cars leave Indianapolis, one traveling east and the other west. After 3 hours they are 297 miles apart. If one car is traveling 5 mph faster than the other, what is the speed of each?

D	= R	· T
3x	x	3
3y	y	3

sum = 297

$y = x+5$

x = rate (speed) of one car

y = rate (speed) of other car

$$\begin{cases}3x+3y=297\\ y=x+5\end{cases}$$

$$\begin{cases}x+y=99\\ x-y=-5\end{cases}$$

$$\begin{bmatrix}1 & 1 & 99\\ 1 & -1 & -5\end{bmatrix} \Rightarrow \begin{bmatrix}1 & 0 & 47\\ 0 & 1 & 52\end{bmatrix}$$

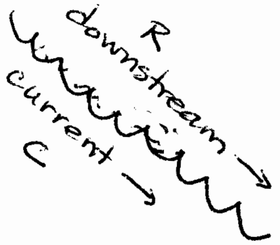
$$\boxed{\begin{array}{l}x = 47 \text{ mph}\\ y = 52 \text{ mph}\end{array}}$$

② Terry can row 10.6 km in 3 hours downstream and 6.8 km upstream in 2 hours. Find how fast Terry can row in still water and the speed of the current.

"Find how fast" → find a rate. R = speed in still water

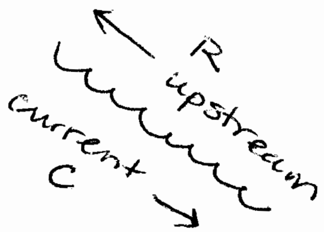
"Find the speed of the current" → find another rate!

C = speed of current



Both rates working together

R+C = effective rate downstream



Rate of rowing is against the rate of current

R−C = effective rate upstream.

D	= R	· T	
10.6 km	R+C	3	downstream
6.8 km	R−C	2	upstream

$$\begin{cases}3(R+C)=10.6\\ 2(R-C)=6.8\end{cases}$$ distribute

$$\begin{cases}3R+3C=10.6\\ 2R-2C=6.8\end{cases}$$ matrix

$$\begin{bmatrix}3 & 3 & 10.6\\ 2 & -2 & 6.8\end{bmatrix}$$ RREF on GC

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$$\left[\begin{array}{ccc} 1 & 0 & 3.4666\ldots \\ 0 & 1 & .0666\ldots \end{array}\right]$$

$\boxed{\text{MATH}}$

1. ▸frac

$$\left[\begin{array}{ccc} 1 & 0 & 52/15 \\ \underset{\underset{R}{\uparrow}}{0} & \underset{\underset{C}{\uparrow}}{1} & 1/15 \end{array}\right] \begin{array}{l} \rightarrow\ 1R + 0C = 52/15 \\ \rightarrow\ 0R + 1C = 1/15 \end{array}$$

interpret matrix

$$\boxed{\begin{array}{ll} R = \frac{52}{15}\ \text{km/hr} & \text{rowing in still water} \\ C = \frac{1}{15}\ \text{km/hr} & \text{speed of current} \end{array}}$$

The instructions did not say to round, so we must give an exact answer. The exact decimals are repeating decimals, so we must use the bar above:

$$\boxed{\begin{array}{ll} R = 3.4\overline{6}\ \text{km/hr} & \text{rowing speed} \\ C = 0.0\overline{6}\ \text{km/hr} & \text{current speed} \end{array}}$$

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3) Lynn Pike, a pharmacist, needs 70 liters of a 50% alcohol solution. She has available a 30% alcohol solution and an 80% alcohol solution. How many liters of each solution should she mix to obtain 70 liters of a 50% solution?

$$\boxed{\begin{matrix}70\text{ L}\\50\%\end{matrix}} = \boxed{\begin{matrix}x\text{ L}\\30\%\end{matrix}} + \boxed{\begin{matrix}y\text{ L}\\80\%\end{matrix}}$$

$$\boxed{\begin{matrix}x = 42\text{ L of } 30\%\\ y = 28\text{ L of } 80\%\end{matrix}}$$

units   part of units

Caution: any use of % must be % TIMES volume

$$\begin{cases} x+y=70 & \leftarrow \text{amt of liquid}\\ .3x+.8y=.5(70) & \leftarrow \text{amt of alcohol in the liquid.} \end{cases}$$

(%) times (volume)

$$\begin{cases} x+y=70\\ .3x+.8y=35 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 70\\ .3 & .8 & 35 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 42\\ 0 & 1 & 28 \end{bmatrix}$$

4) A pharmacist needs 500 ml of a 20% Phenobarbital solution but has only 5% and 25% Phenobarbital solutions available. Find how many ml of each he should mix to get the desired solution.

$$\boxed{\begin{matrix}x\text{ ml}\\5\%\end{matrix}} + \boxed{\begin{matrix}y\text{ ml}\\25\%\end{matrix}} = \boxed{\begin{matrix}500\text{ ml}\\20\%\end{matrix}}$$

$$\begin{cases} x+y=500 & \leftarrow \text{amt of liquid}\\ .05x+.25y=.20(500) \end{cases}$$

amount of Phenobarbital in the liquid

$$\boxed{\begin{matrix}x = 125\text{ ml of } 5\%\\ y = 375\text{ ml of } 25\%\end{matrix}}$$

$$\begin{cases} x+y=500\\ .05x+.25y=100 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 500\\ .05 & .25 & 100 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 125\\ 0 & 1 & 375 \end{bmatrix}$$

total ↓ total ↓ total ↓

5) Rabbits in a lab are kept on a strict daily diet that includes 30 g of protein, 15 g of fat, and 24 g of carbohydrates. The scientist has only three food mixes available with the grams of nutrients per unit given by the table below. Find how many units of each mix are needed daily to meet each rabbit's dietary need. Round to the nearest tenth if necessary.

	protein	fat	carbohydrate
Mix A	4 each unit	6	3
Mix B	6	1	2
Mix C	4	1	12

x = units of food A
y = units of food B
z = units of food C

$x = 1.8$ units of A
$y = 3.1$ units of B
$z = 1.0$ units of C

$$\begin{array}{l} \text{equation of protein:} \\ \text{equation of fat:} \\ \text{equation of carb:} \end{array} \begin{cases} 4x + 6y + 4z = 30 \\ 6x + y + z = 15 \\ 3x + 2y + 12z = 24 \end{cases} \Rightarrow \begin{bmatrix} 4 & 6 & 4 & 30 \\ 6 & 1 & 1 & 15 \\ 3 & 2 & 12 & 24 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 1.81065\ldots \\ 0 & 1 & 0 & 3.10650\ldots \\ 0 & 0 & 1 & 1.02958\ldots \end{bmatrix} \Rightarrow \begin{array}{l} 306/169 \\ 525/169 \\ 174/169 \end{array}$$

↑ exact answers

6) A manufacturing company recently purchased $3000 worth of new equipment to offer new personalized stationery to its customers. The cost of producing a package of personalized stationery is $3.00, and it is sold for $5.50. Find the number of packages that must be sold for the company to break even.

costs = fixed costs + cost of each package times # packages
$y = 3000 + 3x$

y = money
x = # of packages

revenue = price times # packages
$y = 5.5x$

$\boxed{x = 1200 \text{ packages}}$

y = $6600 break-even amount of money

Break-even: revenue = costs

$$\begin{cases} y = 3000 + 3x \\ y = 5.5x \end{cases}$$

$$\begin{bmatrix} 3 & -1 & -3000 \\ 5.5 & -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 1200 \\ 0 & 1 & 6600 \end{bmatrix}$$

7) The measure of the largest angle of a triangle is 80° more than the measure of the smallest angle, and the measure of the remaining angle is 10° more than the measure of the smallest angle. Find the measure of each angle.

largest = 80 + smallest
remain = 10 + smallest

x = smallest
y = largest
z = remaining

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 & -80 \\ 1 & 0 & -1 & -10 \\ 1 & 1 & 1 & 180 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 30 \\ 0 & 1 & 0 & 110 \\ 0 & 0 & 1 & 40 \end{bmatrix}$$

$$\begin{cases} y = 80 + x \\ z = 10 + x \\ x + y + z = 180 \end{cases} \leftarrow \text{angles of } \Delta \text{ add to } 180°$$

$$\boxed{\text{angles are } \begin{array}{r} 30° \\ 110° \\ 40° \end{array}}$$

8) A drafting student bought three templates and a pencil for $6.45, then went back and bought two pads of paper and four pencils for $7.50. If the price of a pad of paper is three times the price of a pencil, find the prices of each type of item.

x = price of template
y = price of pencil
z = price of pads of paper

$$\begin{cases} 3x + y = 6.45 \\ 4y + 2z = 7.5 \\ -3y + z = 0 \end{cases} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1.9 \\ 0 & 1 & 0 & .75 \\ 0 & 0 & 1 & 2.25 \end{bmatrix}$$

$$\begin{array}{l} \text{1st purchase:} \\ \text{2nd:} \\ \text{3rd:} \end{array} \begin{cases} 3x + y = 6.45 \\ 2z + 4y = 7.5 \\ z = 3y \end{cases} \quad \begin{bmatrix} 3 & 1 & 0 & 6.45 \\ 0 & 4 & 2 & 7.5 \\ 0 & -3 & 1 & 0 \end{bmatrix}$$

template costs $1.90
pencil costs $0.75
pad costs $2.25

Extras:

2 variable problems

1. Find how many quarts of 4% butterfat milk and 1% butterfat milk should be mixed to yield 60 quarts of 2% butterfat milk.
2. Karen bought some large frames for \$15 each and some small frames for \$8 each at a closeout sale. If she bought 22 frames for \$239, find how many of each type she bought.
3. An office supply store sells 7 writing tablets and 4 pens for \$6.40. Also, 2 tablets and 19 pens cost \$5.40. Find the price of a tablet and the price of a pen.
4. A candy shop manager mixes M&M's worth \$2 per pound with trail mix worth \$1.50 per pound. Find how man pounds of each she should use to get 50 pounds of a party mix worth \$1.80 per pound.
5. Two cyclists start at the same point and travel in apposite directions. One travels 4 mph faster than the other. In 4 hours they are 112 miles apart. Find how fast each is traveling.
6. Find the break-even point if $C(x) = 105x + 70{,}000$ and $R(x) = 245x$.

3 variable problems:

7. The sum of three numbers is 40. One number is 5 more than a second number. It is also twice the third. Find the numbers.
8. The perimeter of a triangle is 92 cm. If two sides are equally long and the third side is 9 cm longer than the others, find the lengths of the three sides.

① x = #qts of 4%
y = # qts of 1%

$\boxed{\begin{matrix}x \text{ qts}\\ 4\%\end{matrix}} + \boxed{\begin{matrix}y \text{ qts}\\ 1\%\end{matrix}} = \boxed{\begin{matrix}60 \text{ qts}\\ 2\%\end{matrix}}$

$$\begin{cases} x+y=60 \\ .04x+.01y=.02(60) \end{cases}$$

$$\begin{cases} x+y=60 \\ .04x+.01y=1.2 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 60 \\ .04 & .01 & 1.2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 20 \\ 0 & 1 & 40 \end{bmatrix}$$

$\boxed{\begin{matrix}x = 20 \text{ qts of } 4\% \\ y = 40 \text{ qts of } 1\%\end{matrix}}$

② x = # large frames
y = # small frames

$$\begin{cases} x+y=22 & \# \\ 15x+8y=239 & \$ \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 22 \\ 15 & 8 & 239 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 9 \\ 0 & 1 & 13 \end{bmatrix}$$

$\boxed{\begin{matrix}x = 9 \text{ large frames} \\ y = 13 \text{ small frames}\end{matrix}}$

③ x = cost of tablet
y = cost of pen

$$\begin{cases} 7x+4y=6.4 \\ 2x+19y=5.4 \end{cases}$$

$$\begin{bmatrix} 7 & 4 & 6.4 \\ 2 & 19 & 5.4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & .80 \\ 0 & 1 & .20 \end{bmatrix}$$

$\boxed{\begin{matrix}\text{tablets cost } \$0.80 \\ \text{pens cost } \$0.20\end{matrix}}$

④ x = # pounds M&M
y = # lbs of trail mix

$$\begin{cases} x+y=50 & \text{total lbs} \\ 2x+1.5y=50(1.8) & \text{cost} \end{cases}$$

$$\begin{cases} x+y=50 \\ 2x+1.5=90 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 50 \\ 2 & 1.5 & 90 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 20 \end{bmatrix}$$

$\boxed{\begin{matrix}x = 30 \text{ lbs M\&M} \\ y = 20 \text{ lbs trail mix}\end{matrix}}$

⑤

D =	R ·	T
fast 4x	x	4
slow 4y	y	4

$\longleftarrow\bullet\longrightarrow$

opposite directions add distances

$$\begin{cases} 4x+4y=112 \\ x=4y \end{cases}$$

$$\begin{bmatrix} 4 & 4 & 112 \\ 1 & -4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 22.4 \\ 0 & 1 & 5.6 \end{bmatrix}$$

$\boxed{\begin{matrix}x = \text{faster} & 22.4 \text{ mph} \\ y = \text{slower} & 5.6 \text{ mph}\end{matrix}}$

⑥
$$\begin{cases} y = C(x) = 105x + 70000 \\ y = R(x) = 245x \end{cases}$$

$$\begin{cases} 105x - y = -70000 \\ 245x - y = 0 \end{cases}$$

$$\begin{bmatrix} 105 & -1 & -70000 \\ 245 & -1 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 1 & 0 & 500 \\ 0 & 1 & 122500 \end{bmatrix}$$

$\boxed{\begin{matrix}x = 500 \text{ units sold} \\ y = \$122{,}500 \text{ value of goods}\end{matrix}}$

⑦ x, y, z = the numbers

$$\begin{cases} x+y+z=40 \\ x=y+5 \\ x=2z \end{cases}$$

$$\begin{cases} x+y+z=40 \\ x-y \qquad =5 \\ x \qquad -2z=0 \end{cases}$$

$$\begin{bmatrix} 1 & 1 & 1 & 40 \\ 1 & -1 & 0 & 5 \\ 1 & 0 & -2 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 18 \\ 0 & 1 & 0 & 13 \\ 0 & 0 & 1 & 9 \end{bmatrix}$$

$\boxed{\text{The numbers are } 18, 13, 9}$

⑧ x, y, z = sides of triangle

perimeter
$$\begin{cases} x+y+z=92 \\ x=y \\ z=9+x \end{cases}$$

⇒

$$\begin{cases} x + y + z = 92 \\ x - y \qquad = 0 \\ x \qquad - z = -9 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 1 & 92 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & -9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 27.66\ldots \\ 0 & 1 & 0 & 27.66\ldots \\ 0 & 0 & 1 & 36.66\ldots \end{bmatrix}$$

OR improper fractions:

$$\begin{bmatrix} 1 & 0 & 0 & 83/3 \\ 0 & 1 & 0 & 83/3 \\ 0 & 0 & 1 & 110/3 \end{bmatrix}$$

OR mixed numbers

$$\begin{bmatrix} 1 & 0 & 0 & 27\,2/3 \\ 0 & 1 & 0 & 27\,2/3 \\ 0 & 0 & 1 & 36\,2/3 \end{bmatrix}$$

$$\boxed{\begin{array}{ll} \text{sides} & 27.\overline{6}\text{ cm} \\ & 27.\overline{6}\text{ cm} \\ & 36.\overline{6}\text{ cm} \end{array}}$$

← repeat bars required to give exact answers.

OR

$$\boxed{\begin{array}{ll} \text{sides} & 27\,2/3\text{ cm} \\ & 27\,2/3\text{ cm} \\ & 36\,2/3\text{ cm} \end{array}}$$

OR

$$\boxed{\begin{array}{ll} \text{sides} & 83/3\text{ cm} \\ & 83/3\text{ cm} \\ & 110/3\text{ cm} \end{array}}$$

4.3.25 Primo car rental agency charges \$41 per day plus \$0.20 per mile. Ultimo car rental agency charges \$23 per day plus \$0.80 per mile. Find the daily mileage for which the Ultimo charge is three times the Primo charge.

The mileage is 500.

Cost of one-day rental at Primo:

$y = .2x + 41$

$x = \#$miles
$y = $ cost

Cost of one-day rental at Ultimo:

$y = .8x + 23$

Ultimo = 3 times Primo

$$.8x + 23 = 3(.2x + 41)$$
$$.8x + 23 = .6x + 123$$
$$.2x = 100$$
$$\boxed{x = 500 \text{ miles}}$$